

# Nilpotent chiral superfield in $\mathcal{N} = 2$ supergravity and partial rigid supersymmetry breaking

Sergei M. Kuzenko<sup>a</sup> and Gabriele Tartaglino-Mazzucchelli<sup>b</sup>

<sup>a</sup>*School of Physics M013, The University of Western Australia  
35 Stirling Highway, Crawley W.A. 6009, Australia*

<sup>b</sup>*Instituut voor Theoretische Fysica, KU Leuven,  
Celestijnenlaan 200D, B-3001 Leuven, Belgium*

sergei.kuzenko@uwa.edu.au, Gabriele.Tartaglino-Mazzucchelli@fys.kuleuven.be

## Abstract

In the framework of  $\mathcal{N} = 2$  conformal supergravity in four dimensions, we introduce a nilpotent chiral superfield suitable for the description of partial supersymmetry breaking in maximally supersymmetric spacetimes. As an application, we construct Maxwell-Goldstone multiplet actions for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking on  $\mathbb{R} \times S^3$ ,  $\text{AdS}_3 \times S^1$  (or its covering  $\text{AdS}_3 \times \mathbb{R}$ ), and a pp-wave spacetime. In each of these cases, the action coincides with a unique curved-superspace extension of the  $\mathcal{N} = 1$  supersymmetric Born-Infeld action, which is singled out by the requirement of U(1) duality invariance.

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## 1 Introduction

Inspired by the work of Antoniadis, Partouche and Taylor [1], Bagger and Galperin [2] constructed the Goldstone-Maxwell multiplet model for partially broken  $\mathcal{N} = 2$  Poincaré supersymmetry in four spacetime dimensions (4D). Their model proved to coincide with the  $\mathcal{N} = 1$  supersymmetric Born-Infeld action [3, 4]. Two years later, Roček and Tseytlin [5] re-derived the model of [2] using  $\mathcal{N} = 2$  superfields, building on the earlier formulation

due to Roček [6] for the Volkov-Akulov Goldstino model [7] in terms of a nilpotent  $\mathcal{N} = 1$  chiral superfield.<sup>1</sup>

The  $\mathcal{N} = 2$  Minkowski superspace is one of many maximally supersymmetric backgrounds in 4D  $\mathcal{N} = 2$  off-shell supergravity. Such superspaces were classified in [10] building on the earlier analysis [11] of maximally supersymmetric backgrounds in 5D  $\mathcal{N} = 1$  off-shell supergravity. The construction in [5] is down-to-earth in the sense that it is specifically designed to describe the partial breaking of  $\mathcal{N} = 2$  Poincaré supersymmetry. Here we present a theoretical scheme which is suitable for the description of partial supersymmetry breaking in curved maximally supersymmetric backgrounds in 4D  $\mathcal{N} = 2$  off-shell supergravity. As an application of this scheme, we construct Maxwell-Goldstone multiplet actions for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking on  $\mathbb{R} \times S^3$ ,  $\text{AdS}_3 \times S^1$  (or its covering  $\text{AdS}_3 \times \mathbb{R}$ ), and a pp-wave.

This paper is organised as follows. In section 2 we introduce a nilpotent chiral superfield coupled to  $\mathcal{N} = 2$  conformal supergravity. In section 3 we explain how such a superfield can be used to construct a model for partially broken supersymmetry for certain maximally supersymmetric backgrounds of  $\mathcal{N} = 2$  supergravity. The formalism developed is applied in section 4 to re-derive the Roček-Tseytlin construction. In section 5 we construct Maxwell-Goldstone multiplet actions for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking on  $\mathbb{R} \times S^3$ ,  $\text{AdS}_3 \times S^1$  (or its covering  $\text{AdS}_3 \times \mathbb{R}$ ), and a pp-wave. Concluding comments are given in section 6. The main body of the paper is accompanied by three technical appendices. In Appendices A and B, we present group-theoretic formulations for four-dimensional  $\mathcal{N} = 1$  and  $\mathcal{N} = 2$  superspaces over  $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$ . The maximally  $\mathcal{N} = 2$  supersymmetric background over  $\mathbb{R} \times S^3$ , which is used in section 5, is the universal covering space of the  $\mathcal{N} = 2$  superspace over  $(S^1 \times S^3)/\mathbb{Z}_2$ . Appendix A also contains the group-theoretic description of  $\mathcal{N} = 1$  superspace over  $\text{U}(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$ . Appendix C is devoted to the discussion of a unique feature of the anti-de Sitter supersymmetry that distinguishes  $\text{AdS}_4$  from the other maximally supersymmetric four-dimensional backgrounds.

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<sup>1</sup>The same nilpotent chiral superfield was independently introduced, a few months later, by Ivanov and Kapustnikov [8] as a simple application of the general relationship between linear and nonlinear realisations of supersymmetry established in their earlier work [9].

## 2 Nilpotent chiral superfield in $\mathcal{N} = 2$ supergravity

In the framework of four-dimensional  $\mathcal{N} = 2$  conformal supergravity<sup>2</sup> we introduce a nilpotent chiral superfield constrained by

$$\bar{\mathcal{D}}_{\dot{\alpha}}^i \mathcal{Z} = 0 , \quad (2.1a)$$

$$(\mathcal{D}^{ij} + 4S^{ij})\mathcal{Z} - (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\mathcal{Z}} = 4i G^{ij} , \quad (2.1b)$$

$$\mathcal{Z}^2 = 0 , \quad (2.1c)$$

where  $G^{ij}$  is a linear multiplet constrained by  $G^{ij}G_{ij} \neq 0$ . One may interpret  $G^{ij}$  as the field strength of a tensor multiplet. The constraints (2.1a)–(2.1c) are invariant under the  $\mathcal{N} = 2$  super-Weyl transformations [12, 13] if  $\mathcal{Z}$  is considered to be a primary superfield of dimension 1.

A chiral superfield constrained by (2.1b) was considered in [14] in the context of the dilaton effective action in  $\mathcal{N} = 2$  supergravity. In the super-Poincaré case, chiral superfields obeying the constraint (2.1b) with a constant  $G^{ij}$  naturally originate in the framework of partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking [1, 15, 16].

We recall that the  $\mathcal{N} = 2$  tensor multiplet is described in curved superspace by its gauge invariant field strength  $G^{ij}$  which is a linear multiplet. The latter is defined to be a real  $\text{SU}(2)$  triplet (that is,  $G^{ij} = G^{ji}$  and  $\bar{G}_{ij} := \overline{G^{ij}} = G_{ij}$ ) subject to the covariant constraints [17, 18]

$$\mathcal{D}_{\alpha}^{(i} G^{jk)} = \bar{\mathcal{D}}_{\dot{\alpha}}^{(i} G^{jk)} = 0 . \quad (2.2)$$

These constraints are solved in terms of a chiral prepotential  $\Psi$  [19, 20, 21, 22] via

$$G^{ij} = \frac{1}{4}(\mathcal{D}^{ij} + 4S^{ij})\Psi + \frac{1}{4}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Psi} , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i \Psi = 0 , \quad (2.3)$$

which is invariant under Abelian gauge transformations

$$\delta_{\Lambda} \Psi = i\Lambda , \quad (2.4)$$

with  $\Lambda$  a reduced chiral superfield,

$$\bar{\mathcal{D}}_{\dot{\alpha}}^i \Lambda = 0 , \quad (2.5a)$$

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<sup>2</sup>In this paper, we use Howe's superspace formulation [12] for  $\mathcal{N} = 2$  conformal supergravity and follow the supergravity notation and conventions of [13]. In particular, the superspace covariant derivatives are denoted  $\mathcal{D}_{\mathcal{A}} = (\mathcal{D}_a, \mathcal{D}_{\alpha}^i, \bar{\mathcal{D}}_{\dot{\alpha}}^i)$ . We make use of the second-order differential operators  $\mathcal{D}^{ij} := \mathcal{D}^{\alpha(i} \mathcal{D}_{\alpha}^{j)}$ ,  $\bar{\mathcal{D}}^{ij} := \bar{\mathcal{D}}^{\dot{\alpha}(i} \bar{\mathcal{D}}_{\dot{\alpha}}^{j)}$ . The  $\text{SU}(2)$  triplet  $S^{ij} = S^{ji}$  and its conjugate  $\bar{S}_{ij} = \overline{S^{ij}}$  stand for certain components of the superspace torsion tensor.

$$(\mathcal{D}^{ij} + 4S^{ij})\Lambda - (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Lambda} = 0 . \quad (2.5b)$$

We recall that the field strength of an Abelian vector multiplet is a reduced chiral superfield [23].

The constraints on  $\Lambda$  can be solved in terms of the Mezincescu prepotential [24] (see also [19]),  $U_{ij} = U_{ji}$ , which is an unconstrained real  $\mathbf{SU}(2)$  triplet. The curved-superspace solution is [25]

$$\Lambda = \frac{1}{4}\bar{\Delta}(\mathcal{D}^{ij} + 4S^{ij})U_{ij} . \quad (2.6)$$

Here  $\bar{\Delta}$  denotes the chiral projection operator [26, 27]

$$\begin{aligned} \bar{\Delta} &= \frac{1}{96} \left( (\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij})\bar{\mathcal{D}}_{ij} - (\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} + 16\bar{Y}^{\dot{\alpha}\dot{\beta}})\bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}} \right) \\ &= \frac{1}{96} \left( \bar{\mathcal{D}}_{ij}(\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij}) - \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}}(\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} + 16\bar{Y}^{\dot{\alpha}\dot{\beta}}) \right) , \end{aligned} \quad (2.7)$$

with  $\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} := \bar{\mathcal{D}}_k^{(\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\beta})k}$ . Its main properties can be formulated using a super-Weyl inert scalar  $V$ . It holds that

$$\bar{\mathcal{D}}_i^{\dot{\alpha}}\bar{\Delta}V = 0 , \quad (2.8a)$$

$$\delta_\sigma V = 0 \implies \delta_\sigma \bar{\Delta}V = 2\sigma \bar{\Delta}V , \quad (2.8b)$$

$$\int d^4x d^4\theta d^4\bar{\theta} E V = \int d^4x d^4\theta \mathcal{E} \bar{\Delta}V , \quad (2.8c)$$

where the real unconstrained parameter  $\sigma$  corresponds to the super-Weyl transformations [13].<sup>3</sup> Here  $E$  and  $\mathcal{E}$  denote the full superspace and chiral densities, respectively.

The constraints (2.1a) and (2.1b) define a deformed reduced chiral superfield. These constraints may be re-cast in the language of superforms as  $dF = H$ , where  $F$  is a two-form and  $H$  is the three-form field strength,  $dH = 0$ , describing the tensor multiplet [27], see also [28].<sup>4</sup> Switching  $H$  off,  $H = 0$ , turns  $F$  into the two-form field strength of the vector multiplet.

The constraint (2.1b) naturally originates as follows. Consider the model for a massive improved tensor multiplet coupled to  $\mathcal{N} = 2$  conformal supergravity [29, 30]. The action of this model in the form given in [25] is

$$S_{\text{tensor}} = - \int d^4x d^4\theta \mathcal{E} \left\{ \Psi \mathbb{W} + \frac{1}{4}\mu(\mu + ie)\Psi^2 \right\} + \text{c.c.} , \quad (2.9)$$

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<sup>3</sup>The parameter  $\sigma$  was denoted  $2U$  in [13].

<sup>4</sup>We are grateful to Joseph Novak for this observation.

where  $\mu$  and  $e$  are real parameters, with  $\mu \neq 0$  (the tensor multiplet mass can be shown to be  $m = \sqrt{\mu^2 + e^2}$ ). The kinetic term involves the composite [31]

$$\mathbb{W} := -\frac{G}{8}(\bar{\mathcal{D}}_{ij} + 4\bar{S}_{ij}) \left( \frac{G^{ij}}{G^2} \right) , \quad (2.10)$$

which proves to be a reduced chiral superfield.<sup>5</sup> For  $m = 0$  the above action describes the improved tensor multiplet [31]. We introduce a Stückelberg-type extension of the model

$$\tilde{S}_{\text{tensor}} = - \int d^4x d^4\theta \mathcal{E} \left\{ \Psi \mathbb{W} + \frac{1}{4} \mu (\mu + ie)(\Psi - iW)^2 \right\} + \text{c.c.} , \quad (2.11)$$

where  $W$  is the field strength of a vector multiplet. The action is invariant under the gauge transformation (2.4) accompanied by

$$\delta_\Lambda W = \Lambda . \quad (2.12)$$

The original action (2.9) is obtained from (2.11) by choosing a gauge  $W = 0$ . Now one can see that the superfield  $\mathcal{Z} := W + i\Psi$  obeys the constraint (2.1b).

It is well known that the functional

$$i \int d^4x d^4\theta \mathcal{E} W^2 + \text{c.c.} \quad (2.13)$$

is a total derivative. Since the mass term in (2.11) is invariant under the gauge transformation (2.4) and (2.12), it follows that, given a chiral superfield  $\mathcal{Z}$  constrained by (2.1b), the functional

$$I = \int d^4x d^4\theta \mathcal{E} \left\{ \mathcal{Z} \Psi - \frac{i}{2} \Psi^2 \right\} + \text{c.c.} \quad (2.14)$$

is invariant under the gauge transformation (2.4),  $\delta_\Lambda I = 0$ .

The constraints (2.1a)–(2.1c) imply that, for certain supergravity backgrounds, the degrees of freedom described by the  $\mathcal{N} = 2$  chiral superfield  $\mathcal{Z}$  are in a one-to-one correspondence with those of an Abelian  $\mathcal{N} = 1$  vector multiplet. The specific feature of such  $\mathcal{N} = 2$  supergravity backgrounds is that they possess an  $\mathcal{N} = 1$  subspace of the full  $\mathcal{N} = 2$  superspace. This property is not universal. In particular, there exist maximally  $\mathcal{N} = 2$  supersymmetric backgrounds with no admissible truncation to  $\mathcal{N} = 1$  [10].

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<sup>5</sup>The superfield (2.10) is one of the simplest applications of the powerful approach to generate composite reduced chiral multiplets which was presented in [25].

### 3 Maximally $\mathcal{N} = 2$ supersymmetric backgrounds and partial supersymmetry breaking

So far we have discussed an arbitrary supergravity background. Now we restrict our consideration to a maximally supersymmetric background  $\mathbb{M}^{4|8}$  with the property that the chiral prepotential  $\Psi$  for  $G^{ij}$  may be chosen such that the following two conditions hold. Firstly, the complex linear multiplet

$$G_+^{ij} := \frac{1}{4}(\mathcal{D}^{ij} + 4S^{ij})\Psi \quad (3.1)$$

is covariantly constant and null,

$$\mathcal{D}_A G_+^{ij} = 0 \ , \quad (3.2)$$

$$G_+^{ij} G_{+ij} = 0 \ . \quad (3.3)$$

Secondly, the prepotential  $\Psi$  may be chosen to be nilpotent,

$$\Psi^2 = 0 \ . \quad (3.4)$$

The null condition for  $G_+^{ij}$  means that  $G_+^{ij} = q^i q^j$ , for some isospinor  $q^i$ . It follows that  $G^{ij} = G_+^{ij} + G_-^{ij}$  is covariantly constant,

$$\mathcal{D}_A G^{ij} = 0 \ , \quad (3.5)$$

where we have denoted  $G_-^{ij} := \frac{1}{4}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Psi}$ .

We are going to show that the following functional

$$I = \int d^4x d^4\theta \mathcal{E} \Psi \mathcal{Z} \quad (3.6)$$

is supersymmetric. Here  $\mathcal{Z}$  is the nilpotent chiral superfield (2.1), which is assumed to be a composite of the dynamical fields. The complex linear multiplet (3.1) and its chiral prepotential  $\Psi$  are background fields associated with the background superspace  $\mathbb{M}^{4|8}$ . Since the covariant derivatives  $\mathcal{D}_A$  are invariant under the isometry transformations of  $\mathbb{M}^{4|8}$ , the fields  $G_+^{ij}$  and  $\Psi$  do not change under such transformations. Let  $\xi$  be a Killing supervector field for  $\mathbb{M}^{4|8}$  (see section 6.4 of [32] and [33] for general discussions). Then

$$\delta_\xi I = \int d^4x d^4\theta \mathcal{E} \Psi \delta_\xi \mathcal{Z} = - \int d^4x d^4\theta \mathcal{E} \mathcal{Z} \delta_\xi \Psi \ . \quad (3.7)$$

We introduce a reduced chiral superfield  $W$  by

$$\mathcal{Z} = W + \mathrm{i}\Psi , \quad W = \frac{1}{4}\bar{\Delta}\left(\mathcal{D}^{ij} + 4S^{ij}\right)U_{ij} , \quad (3.8)$$

where  $U_{ij}$  is the Mezincescu prepotential for the reduced chiral superfield  $W$ . Since  $\Psi\delta_\xi\Psi = 0$ , we have

$$\begin{aligned} \delta_\xi I &= - \int \mathrm{d}^4x \mathrm{d}^4\theta \mathcal{E} \mathcal{Z} \delta_\xi \Psi = - \int \mathrm{d}^4x \mathrm{d}^4\theta \mathcal{E} W \delta_\xi \Psi \\ &= -\frac{1}{4} \int \mathrm{d}^4x \mathrm{d}^4\theta \mathrm{d}^4\bar{\theta} E U_{ij} \left(\mathcal{D}^{ij} + 4S^{ij}\right) \delta_\xi \Psi \\ &= -\frac{1}{4} \int \mathrm{d}^4x \mathrm{d}^4\theta \mathrm{d}^4\bar{\theta} E U_{ij} \delta_\xi \left(\mathcal{D}^{ij} + 4S^{ij}\right) \Psi \\ &= - \int \mathrm{d}^4x \mathrm{d}^4\theta \mathrm{d}^4\bar{\theta} E U_{ij} \delta_\xi G_+^{ij} = 0 . \end{aligned} \quad (3.9)$$

In the next two sections, it will be shown that the action

$$S = -\frac{\mathrm{i}}{4} \int \mathrm{d}^4x \mathrm{d}^4\theta \mathcal{E} \Psi \mathcal{Z} + \text{c.c.} \quad (3.10)$$

describes the Maxwell-Goldstone multiplet for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking on the maximally supersymmetric backgrounds specified.

The above derivation does not use the null condition (3.3). The latter is introduced for the  $\mathcal{N} = 2$  superspace  $\mathbb{M}^{4|8}$  to possess an  $\mathcal{N} = 1$  subspace.

## 4 Example: The super-Poincaré case

The simplest maximally supersymmetric background is  $\mathcal{N} = 2$  Minkowski superspace. In this superspace, every constant real  $\mathrm{SU}(2)$  triplet  $G^{ij}$  is covariantly constant,

$$D_{\mathcal{A}} G^{ij} = 0 , \quad (4.1)$$

where  $D_{\mathcal{A}} = (\partial_a, D_\alpha^i, \bar{D}_{\dot{\alpha}}^i)$  are the flat superspace covariant derivatives. Let  $\Psi$  be a chiral prepotential for  $G^{ij}$ ,  $\bar{D}_{\dot{\alpha}}^i \Psi = 0$ . We represent

$$G^{ij} = G_+^{ij} + G_-^{ij} , \quad G_+^{ij} = \frac{1}{4} D^{ij} \Psi , \quad G_-^{ij} = \frac{1}{4} \bar{D}^{ij} \bar{\Psi} . \quad (4.2)$$

It is always possible to choose the prepotential  $\Psi$  such that the following properties hold:

$$\Psi^2 = 0 , \quad D_{\mathcal{A}} G_+^{ij} = 0 , \quad G_+^{ij} G_{+ij} = 0 . \quad (4.3)$$



In  $\mathcal{N} = 2$  Minkowski superspace, the constraints (2.1a)–(2.1c) turn into

$$\bar{D}_\alpha^i \mathcal{Z} = 0 , \quad (4.4a)$$

$$D^{ij} \mathcal{Z} - \bar{D}^{ij} \bar{\mathcal{Z}} = 4i G^{ij} , \quad (4.4b)$$

$$\mathcal{Z}^2 = 0 . \quad (4.4c)$$

The action (3.10) becomes

$$S = -\frac{i}{4} \int d^4x d^4\theta \mathcal{Z} \Psi + \text{c.c.} \quad (4.5)$$

Since  $G_+^{ij}$  is constant, it is invariant under the  $\mathcal{N} = 2$  supersymmetry transformations. In accordance with the analysis given in the previous section, the action is  $\mathcal{N} = 2$  supersymmetric.

For the Grassmann coordinates  $\theta_i^\alpha$  and  $\bar{\theta}_\alpha^i$  of  $\mathcal{N} = 2$  Minkowski superspace, as well as for the spinor covariant derivatives  $D_\alpha^i$  and  $\bar{D}_i^\alpha$ , it is useful to label the values of their  $R$ -symmetry indices as  $i, j = \underline{1}, \underline{2}$ . Without loss of generality we can choose

$$G_+^{ij} = -i\delta_2^i \delta_2^j , \quad \Psi = i\theta_2^\alpha \theta_{\alpha 2} . \quad (4.6)$$

We can now reproduce the results of [2] from the  $\mathcal{N} = 2$  setup described. In order to solve the constraints (4.4), it is useful to carry out a reduction to  $\mathcal{N} = 1$  Minkowski superspace.

Given a superfield  $U(x, \theta_i, \bar{\theta}^i)$  on  $\mathcal{N} = 2$  Minkowski superspace, we introduce its bar-projection

$$U| := U(x, \theta_i, \bar{\theta}^i)|_{\theta_2 = \bar{\theta}_2 = 0} , \quad (4.7)$$

which is a superfield on  $\mathcal{N} = 1$  Minkowski superspace with the Grassmann coordinates  $\theta^\alpha = \theta_\underline{1}^\alpha$  and  $\bar{\theta}_\alpha = \bar{\theta}_\alpha^{\underline{1}}$  and the spinor covariant derivatives  $D_\alpha = D_\alpha^{\underline{1}}$  and  $\bar{D}^\alpha = \bar{D}_\underline{1}^\alpha$ . The background superfield  $\Psi$  is characterised by the properties

$$\Psi| = 0 , \quad D_\alpha^2 \Psi| = 0 . \quad (4.8)$$

Since  $\mathcal{Z}^2 = 0$ , the constraints (4.4) imply

$$(D^{\alpha 2} \mathcal{Z}) D_\alpha^2 \mathcal{Z} + \mathcal{Z} \bar{D}_{\underline{1}\underline{1}} \bar{\mathcal{Z}} + 4\mathcal{Z} = 0 . \quad (4.9)$$

Taking the bar-projection of this constraint gives

$$X + \frac{1}{4} X \bar{D}^2 \bar{X} = W^2 , \quad W^2 := W^\alpha W_\alpha , \quad (4.10)$$

where we have introduced the  $\mathcal{N} = 1$  components of  $\mathcal{Z}$ :

$$X := \mathcal{Z}|, \quad W_\alpha := -\frac{i}{2}D_\alpha^2 \mathcal{Z}|. \quad (4.11a)$$

These superfields satisfy the constraints

$$\bar{D}_{\dot{\alpha}}X = 0, \quad \bar{D}_{\dot{\alpha}}W_\alpha = 0, \quad D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}. \quad (4.11b)$$

The constraints on  $W_\alpha$  tell us that it can be interpreted as the field strength of an Abelian  $\mathcal{N} = 1$  vector multiplet. The constraint (4.10) is equivalent to the Bagger-Galperin constraint [2]. Its general solution is

$$X = W^2 - \frac{1}{2}\bar{D}^2 \frac{W^2\bar{W}^2}{\left(1 + \frac{1}{2}A + \sqrt{1 + A + \frac{1}{4}B^2}\right)}, \quad (4.12a)$$

$$A = \frac{1}{2}(D^2W^2 + \bar{D}^2\bar{W}^2), \quad B = \frac{1}{2}(D^2W^2 - \bar{D}^2\bar{W}^2). \quad (4.12b)$$

Upon reduction to  $\mathcal{N} = 1$  superspace, the action (4.5) becomes

$$I = \frac{1}{4} \int d^4x d^2\theta X + \frac{1}{4} \int d^4x d^2\bar{\theta} \bar{X}. \quad (4.13)$$

This is the  $\mathcal{N} = 1$  supersymmetric Born-Infeld action. Being manifestly  $\mathcal{N} = 1$  supersymmetric, the action is also invariant under the second nonlinearly realised supersymmetry transformation [2]

$$\delta_\epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4}\epsilon_\alpha \bar{D}^2 \bar{X} + i\bar{\epsilon}^{\dot{\beta}} \partial_{\alpha\dot{\beta}} X \implies \delta_\epsilon X = 2\epsilon^\alpha W_\alpha. \quad (4.14)$$

For completeness, we re-derive this result.

Let  $U$  be a scalar superfield on  $\mathcal{N} = 2$  Minkowski superspace. Its isometry transformation is

$$\delta_\xi U = -\xi U, \quad (4.15)$$

where

$$\xi = \bar{\xi} = \xi^A D_A = \xi^a \partial_a + \xi_i^\alpha D_\alpha^i + \bar{\xi}_{\dot{\alpha}}^{\dot{i}} \bar{D}_{\dot{\alpha}}^{\dot{i}} \quad (4.16)$$

is a Killing supervector field of Minkowski superspace,<sup>6</sup>

$$\xi_i^\alpha = -\frac{i}{8}\bar{D}_{\dot{\beta}i}\xi^{\dot{\beta}\alpha}, \quad D_{(\alpha}^i \xi_{\beta)\dot{\beta}} = \bar{D}_{i(\dot{\alpha}} \xi_{\beta\dot{\beta})} = 0, \quad D_\alpha^i \xi_i^\alpha = 0. \quad (4.17)$$

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<sup>6</sup>It follows from (4.17) that  $\xi_i^\alpha$  is chiral,  $\bar{D}_{\dot{\beta}}^j \xi_i^\alpha = 0$ .

The Killing supervector field generating the supersymmetry transformation is characterised by the components

$$\xi^a = 2i(\theta_i \sigma^a \bar{\epsilon}^i - \epsilon_i \sigma^a \bar{\theta}^i) , \quad \xi_i^\alpha = \epsilon_i^\alpha = \text{const} . \quad (4.18)$$

Applying this transformation to  $\mathcal{Z}$  gives  $\delta_\xi \mathcal{Z} = -(\xi^\alpha \partial_a + \xi_i^\alpha D_\alpha^i) \mathcal{Z}$ . We now consider only the second supersymmetry transformation by choosing  $\epsilon_{\underline{1}}^\alpha = 0$  and  $\epsilon_{\underline{2}}^\alpha = \epsilon^\alpha$ . It acts on the  $\mathcal{N} = 1$  superfields (4.11a) as follows

$$\delta_\epsilon X = \delta_\xi \mathcal{Z} = -(\xi \mathcal{Z}) = -\epsilon^\alpha (D_\alpha^2 \mathcal{Z}) = -2i\epsilon^\alpha W_\alpha , \quad (4.19a)$$

$$\delta_\epsilon W_\alpha = -\frac{i}{2} (D_\alpha^2 \delta_\xi \mathcal{Z}) = -i\epsilon_\alpha - \frac{i}{4} \epsilon_\alpha \bar{D}^2 \bar{X} - \bar{\epsilon}^{\dot{\beta}} \partial_{\alpha\dot{\beta}} X , \quad (4.19b)$$

where we have made use of the constraints obeyed by  $\mathcal{Z}$  and  $X$ . The supersymmetry transformation (4.14) follows from (4.19) upon a rescaling of  $\epsilon^\alpha$ .

## 5 Maxwell-Goldstone multiplet for partially broken rigid supersymmetry in curved space

We turn to applying the theoretical framework of section 3 to maximally supersymmetric curved backgrounds in  $\mathcal{N} = 2$  supergravity.

### 5.1 Curved $\mathcal{N} = 2$ superspace backgrounds

We consider a maximally supersymmetric background  $\mathbb{M}^{4|8}$  described by the following algebra of covariant derivatives<sup>7</sup>

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = \{\bar{\mathcal{D}}_i^{\dot{\alpha}}, \bar{\mathcal{D}}_j^{\dot{\beta}}\} = 0 , \quad (5.1a)$$

$$\begin{aligned} \{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_j^{\dot{\beta}}\} &= -2i\delta_j^i \mathcal{D}_\alpha^{\dot{\beta}} + 4iG^{\gamma\dot{\beta}i}{}_j M_{\alpha\gamma} + 4iG_{\alpha\dot{\gamma}}{}^i{}_j \bar{M}^{\dot{\gamma}\dot{\beta}} \\ &\quad - 4i\delta_j^i G_\alpha{}^{\dot{\beta}kl} J_{kl} - 2iG_\alpha{}^{\dot{\beta}i}{}_j Y , \end{aligned} \quad (5.1b)$$

$$[\mathcal{D}_a, \mathcal{D}_\beta^j] = (\tilde{\sigma}_a)^{\dot{\alpha}\gamma} G_{\beta\dot{\alpha}}{}^j{}_k \mathcal{D}_\gamma^k , \quad [\mathcal{D}_a, \bar{\mathcal{D}}_{\dot{\beta}j}] = -(\tilde{\sigma}_a)^{\dot{\gamma}\alpha} G_{\alpha\dot{\beta}}{}^k{}_j \bar{\mathcal{D}}_{\dot{\gamma}k} , \quad (5.1c)$$

where the torsion tensor  $G_a^{ij}$  is annihilated by the spinor covariant derivatives,

$$\mathcal{D}_\alpha^i G_b^{jk} = 0 , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i G_b^{jk} = 0 . \quad (5.1d)$$

---

<sup>7</sup>Here  $M_{ab}$ ,  $J^{kl}$  and  $Y$  are the Lorentz, SU(2) and U(1) generators, respectively, defined as in [13].

This algebra is obtained from that corresponding to  $\mathcal{N} = 2$  conformal supergravity, and given by eq. (2.8) in [13], by (i) switching off the components  $S^{ij}$ ,  $Y_{\alpha\beta}$ ,  $W_{\alpha\beta}$  and  $G_{\alpha\dot{\alpha}}$  of the torsion tensor; and (ii) imposing (5.1d). The constraints (5.1d) are required by the theorem [11] that all fermionic components of the superspace torsion tensor must vanish in maximally supersymmetric backgrounds.

In complete analogy with the 5D case [11], the constraints (5.1d) imply the following integrability condition

$$G_a{}^{k(i}G_b{}^{j)}{}_k = 0 . \quad (5.2)$$

As shown in [11], the general solution of the conditions (5.1d) and (5.2) is

$$G_b{}^{kl} = -\frac{1}{4}g_b s^{kl} , \quad \mathcal{D}_\alpha^i g_b = 0 , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i g_b = 0 , \quad \mathcal{D}_A s^{kl} = 0 , \quad (5.3)$$

for some real vector  $g_b$  and real  $\text{SU}(2)$  triplet  $s^{kl}$ . The latter may be normalised as

$$s^{ij}s_{ij} = 2 . \quad (5.4)$$

Since  $g^2 = g^a g_a$  is constant,  $\mathcal{D}_A g^2 = 0$ , there are in fact three different superspaces described by the above algebra: (i) if  $g_a$  is time-like,  $g^2 < 0$ , the bosonic body of  $\mathbb{M}^{4|8}$  is  $\mathbb{R} \times S^3$ ; (ii) if  $g_a$  is space-like,  $g^2 > 0$ , the bosonic body of  $\mathbb{M}^{4|8}$  is  $\text{AdS}_3 \times \mathbb{R}$ ; (iii) in the null case,  $g^2 = 0$ , the spacetime geometry is a pp-wave. We will denote these superspaces as  $\mathbb{M}_T^{4|8}$ ,  $\mathbb{M}_S^{4|8}$  and  $\mathbb{M}_N^{4|8}$ , respectively. These backgrounds were constructed in [10], and they have 5D cousins [11].

In order to get some more insight into the structure of the superspace geometry (5.1), a specific value of  $g^2$  has to be fixed. It suffices to consider the superspace  $\mathbb{M}_T^{4|8}$ , since the other two cases may be treated similarly. As a supermanifold,  $\mathbb{M}_T^{4|8}$  is the universal covering of the 4D  $\mathcal{N} = 2$  superspace introduced in Appendix B.

In the case  $g^2 < 0$ , it is possible to choose a Lorentz and  $\text{SU}(2)_R$  gauge such that

$$g_a = (g, 0, 0, 0) , \quad s_i{}^j = -i(\sigma^3)_i{}^j = i(-1)^i \delta_i^j . \quad (5.5)$$

As shown in [10], the algebra of covariant derivatives is equivalent to

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}^i, \bar{\mathcal{D}}_{\dot{\beta}}^j\} = 0 , \quad \{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\dot{\beta}}^j\} = -2i\delta_j^i (\sigma^a)_\alpha{}^{\dot{\beta}} \mathcal{D}_a^{(i)} , \quad (5.6a)$$

$$[\mathcal{D}_a^{(i)}, \mathcal{D}_\beta^j] = \frac{i}{2}\delta^{ij}(-1)^j (\sigma_a)_{\beta\dot{\beta}} g^{\dot{\beta}\gamma} \mathcal{D}_\gamma^j , \quad (5.6b)$$

$$[\mathcal{D}_a^{(i)}, \mathcal{D}_b^{(j)}] = (-1)^{j+1} \delta^{ij} \varepsilon_{abc} g^c \mathcal{D}_d^{(j)} , \quad (5.6c)$$

where we have introduced the “improved” vector covariant derivatives

$$\mathcal{D}_a^{(i)} := \mathcal{D}_a + \frac{1}{2}g_a s^{kl} J_{kl} + (-1)^i \left( \frac{1}{4} \varepsilon_{abcd} g^b M^{cd} + i g_a Y \right) . \quad (5.7)$$

These (anti-)commutation relations correspond to the superalgebra  $\mathfrak{su}(2|1) \times \mathfrak{su}(2|1)$ .

The superspace geometry of  $\mathbb{M}_T^{4|8}$  can be described, e.g., in terms of the covariant derivatives  $\tilde{\mathcal{D}}_{\mathcal{A}} = (\mathcal{D}_a^{(1)}, \mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\dot{\alpha}}^i)$ . In accordance with (5.6), the operators  $(\mathcal{D}_a^{(1)}, \mathcal{D}_\alpha^1, \bar{\mathcal{D}}_{\dot{\alpha}}^1)$  form a closed algebra isomorphic to that of the superalgebra  $\mathfrak{su}(2|1)$ . This property means that the  $\mathcal{N} = 2$  superspace  $\mathbb{M}_T^{4|8}$  possesses an  $\mathcal{N} = 1$  subspace which will be denoted  $\mathbb{M}_T^{4|4}$ . It turns out that all the conditions (3.2)–(3.4) can be met in the case of  $\mathbb{M}_T^{4|8}$ . In particular, this superspace allows the existence of covariantly constant complex  $\text{SU}(2)$  triplets  $G_+^{ij}$ . Since the graded commutation relations for  $\tilde{\mathcal{D}}_{\mathcal{A}}$  involve neither Lorentz nor  $\text{SU}(2)$  curvature tensors, the Lorentz and  $\text{SU}(2)$  connections may be gauged away. In such a gauge, every constant complex  $\text{SU}(2)$  triplets  $G_+^{ij}$  is covariantly constant.

Since the superspaces  $\mathbb{M}_T^{4|8}$ ,  $\mathbb{M}_S^{4|8}$  and  $\mathbb{M}_N^{4|8}$  meet the requirements (3.2)–(3.4), the formalism of section 3 may be used to construct a Maxwell-Goldstone multiplet action for partial supersymmetry breaking. Instead of implementing the scheme directly, we will take a shortcut to constructing such actions on the  $\mathcal{N} = 1$  subspaces of the superspaces  $\mathbb{M}_T^{4|8}$ ,  $\mathbb{M}_S^{4|8}$  and  $\mathbb{M}_N^{4|8}$ .

## 5.2 Goldstone multiplet for partially broken supersymmetry

We consider a maximally supersymmetric background  $\mathbb{M}^{4|4}$  described by the following algebra of  $\mathcal{N} = 1$  covariant derivatives:

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0 , \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0 , \quad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i \mathcal{D}_{\alpha\dot{\beta}} , \quad (5.8a)$$

$$[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot{\beta}}] = i \varepsilon_{\alpha\beta} G_{\dot{\beta}}^\gamma \mathcal{D}_\gamma , \quad [\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i \varepsilon_{\dot{\alpha}\dot{\beta}} G_{\beta}^{\dot{\gamma}} \bar{\mathcal{D}}_{\dot{\gamma}} , \quad (5.8b)$$

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i \varepsilon_{\dot{\alpha}\dot{\beta}} G_{\beta}^{\dot{\gamma}} \mathcal{D}_{\alpha\dot{\gamma}} + i \varepsilon_{\alpha\beta} G_{\dot{\beta}}^\gamma \mathcal{D}_{\gamma\dot{\alpha}} , \quad (5.8c)$$

where the torsion tensor  $G_a$  is covariantly constant,

$$\mathcal{D}_A G_b = 0 . \quad (5.8d)$$

This is a special case of the superspace geometry for  $\mathcal{N} = 1$  old minimal supergravity [34] reviewed in [32]. The above algebra is obtained from the supergravity (anti-)commutation relations (5.5.6) and (5.5.7) in [32] by (i) switching off the chiral torsion superfields  $R$  and  $W_{\alpha\beta\gamma}$  and their conjugates; and (ii) imposing the condition (5.8d).

Since  $G^2 = G^a G_a$  is constant, the geometry (5.8) describes three different superspaces,  $\mathbb{M}_T^{4|4}$ ,  $\mathbb{M}_S^{4|4}$  and  $\mathbb{M}_N^{4|4}$ , which correspond to the choices  $G^2 < 0$ ,  $G^2 > 0$  and  $G^2 = 0$ , respectively. These  $\mathcal{N} = 1$  superspaces originate as the  $\mathcal{N} = 1$  subspaces of the  $\mathcal{N} = 2$  superspaces of  $\mathbb{M}_T^{4|8}$ ,  $\mathbb{M}_S^{4|8}$  and  $\mathbb{M}_N^{4|8}$ , respectively, considered in the previous subsection.<sup>8</sup> We recall that the Lorentzian manifolds supported by these superspaces are  $\mathbb{R} \times S^3$ ,  $\text{AdS}_3 \times S^1$  or its covering  $\text{AdS}_3 \times \mathbb{R}$ , and a pp-wave spacetime, respectively.<sup>9</sup> As a supermanifold,  $\mathbb{M}_T^{4|4}$  is the universal covering of the  $\mathcal{N} = 1$  superspace  $\mathcal{M}^{4|4}$  introduced in section A.1. The isometry group of  $\mathcal{M}^{4|4}$  is  $\text{SU}(2|1) \times \text{U}(2)$ . As a supermanifold,  $\mathbb{M}_S^{4|4}$  is the universal covering of the  $\mathcal{N} = 1$  superspace  $\widetilde{\mathcal{M}}^{4|4}$  introduced in section A.2. The isometry group of  $\widetilde{\mathcal{M}}^{4|4}$  is  $\text{SU}(1, 1|1) \times \text{U}(2)$ .

The superspace  $\mathbb{M}^{4|4}$  allows the existence of covariantly constant spinors,

$$\mathcal{D}_A \epsilon_\alpha = 0 . \quad (5.9)$$

Such a spinor is constant in a gauge in which the Lorentz connection vanishes.

By analogy with the flat-superspace case, we consider the following  $\mathcal{N} = 1$  supersymmetric theory with action

$$S = \frac{1}{4} \int d^4x d^2\theta \mathcal{E} X + \text{c.c.} , \quad (5.10)$$

where the covariantly chiral superfield  $X$  is a unique solution of the constraint

$$X + \frac{1}{4} X \bar{\mathcal{D}}^2 \bar{X} = W^2 . \quad (5.11)$$

The superfield  $W_\alpha$  is the chiral field strength of an Abelian vector multiplet and, together with its complex conjugate  $\bar{W}_{\dot{\alpha}}$ , it obeys the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} . \quad (5.12)$$

The explicit solution of the constraint (5.11) is a covariantisation of that described in the previous section. It is given, e.g., in [38].

The action (5.10) is invariant under a second supersymmetry given by

$$\delta_\epsilon X = 2\epsilon^\alpha W_\alpha , \quad (5.13)$$

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<sup>8</sup>In the time-like case,  $G^2 < 0$ , the graded commutation relations (5.8) are obtained from (5.6) by choosing  $i, j = \underline{1}$  and setting  $G_a = g_a$ .

<sup>9</sup> $\mathcal{N} = 1$  supersymmetric theories on  $\mathbb{R} \times S^3$  were studied in the mid-1980s by Sen [35]. At the component level, the maximally  $\mathcal{N} = 1$  supersymmetric backgrounds in four dimensions were classified by Festuccia and Seiberg [36]. Their results were re-derived in [37] using the superspace formalism developed in the mid-1990s [32].

with the parameter  $\epsilon_\alpha$  being constrained as in (5.9). Of course, this transformation should be induced by that of  $W_\alpha$ . The correct supersymmetry transformation of  $W_\alpha$  proves to be

$$\delta_\epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4}\epsilon_\alpha \bar{\mathcal{D}}^2 \bar{X} + i\bar{\epsilon}^{\dot{\beta}} \mathcal{D}_{\alpha\dot{\beta}} X - \bar{\epsilon}^{\dot{\beta}} G_{\alpha\dot{\beta}} X . \quad (5.14)$$

It has the correct flat superspace limit [2], compare with (4.14), and respects the Bianchi identity (5.12),

$$\mathcal{D}^\alpha \delta_\epsilon W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \delta_\epsilon \bar{W}^{\dot{\alpha}} . \quad (5.15)$$

The dynamical system defined by eqs. (5.10) and (5.11) describes the Maxwell-Goldstone multiplet action for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking in those curved spacetimes which are supported by the superspace geometry (5.8), including  $\mathbb{R} \times S^3$ ,  $\text{AdS}_3 \times S^1$  and its covering  $\text{AdS}_3 \times \mathbb{R}$ .

## 6 Concluding comments

There are five types of maximally supersymmetric backgrounds in four-dimensional  $\mathcal{N} = 1$  off-shell supergravity, two of which are well known: Minkowski superspace  $\mathbb{R}^{4|4}$  [39, 40] and anti-de Sitter superspace  $\text{AdS}^{4|4}$  [41, 42, 43]. The remaining three superspaces,  $\mathbb{M}_T^{4|4}$ ,  $\mathbb{M}_S^{4|4}$  and  $\mathbb{M}_N^{4|4}$ , are described by the geometry (5.8) with different choices of  $G_a$ . All five  $\mathcal{N} = 1$  superspaces possess  $\mathcal{N} = 2$  extensions. The Maxwell-Goldstone multiplet on  $\mathbb{R}^{4|4}$  for partially broken  $\mathcal{N} = 2$  Poincaré supersymmetry was found long ago [2, 5]. In this paper, we have constructed the Maxwell-Goldstone multiplets which are defined on  $\mathbb{M}_T^{4|4}$ ,  $\mathbb{M}_S^{4|4}$  and  $\mathbb{M}_N^{4|4}$  and describe partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking.

In Appendix C we demonstrate that no Maxwell-Goldstone multiplet *action* for partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking exists in the case of the anti-de Sitter (AdS) supersymmetry. The reason for this obstruction is the fact that every covariantly constant  $\text{SU}(2)$  triplet  $G_+^{ij}$  must be proportional to the torsion tensor  $S^{ij}$ , which is real and covariantly constant in  $\text{AdS}^{4|8}$  [44]. As a consequence, the conditions (3.2) and (3.3) are not compatible in  $\text{AdS}^{4|8}$ . Since the  $\mathcal{N} = 1$  AdS superspace  $\text{AdS}^{4|4}$  is naturally embedded in  $\text{AdS}^{4|8}$  as a subspace [73], applying the formalism of section 2 to the case of  $\text{AdS}^{4|8}$  allows us to derive a Maxwell-Goldstone multiplet for partially broken  $\mathcal{N} = 2$  AdS supersymmetry. The corresponding technical details are spelled out in Appendix C. However, since the conditions (3.2) and (3.3) are not compatible in  $\text{AdS}^{4|8}$ , we cannot use this Maxwell-Goldstone multiplet to construct a supersymmetric invariant action.

There exists a one-parameter family of  $\mathcal{N} = 1$  supersymmetric extensions of the Born-Infeld actions [4]. A unique extension is fixed by the requirement that the action should describe the Maxwell-Goldstone multiplet on  $\mathbb{R}^{4|4}$  for partially broken  $\mathcal{N} = 2$  Poincaré supersymmetry [2, 5]. The same extension is uniquely fixed by the requirement of  $U(1)$  duality invariance [45, 46], which implies the self-duality under superfield Legendre transform discovered by Bagger and Galperin [2]. A curved-superspace extension of the  $\mathcal{N} = 1$  supersymmetric Born-Infeld action is not unique. However, a unique extension is fixed by the requirement of  $U(1)$  duality invariance [38]. It is given by the action (5.10) in which  $X$  is a unique solution to the constraint

$$X + \frac{1}{4}X(\bar{\mathcal{D}}^2 - 4R)\bar{X} = W^2, \quad (6.1)$$

with  $R$  the chiral scalar torsion superfield. This action was first proposed in [47]. In the case of anti-de Sitter superspace  $\text{AdS}^{4|4}$ , the only non-zero components of the superspace torsion are  $R$  and  $\bar{R}$ , which are constant. The corresponding  $\mathcal{N} = 1$  supersymmetric Born-Infeld action possesses  $U(1)$  duality invariance, however it is not invariant under a second nonlinearly realised supersymmetry, as demonstrated in Appendix C. Therefore, this action is not suitable to describe a partial breaking of the  $\mathcal{N} = 2$  AdS supersymmetry.

In addition to the Maxwell-Goldstone multiplet of [2, 5], there exist other multiplets for partially broken  $\mathcal{N} = 2$  Poincaré supersymmetry [48, 5, 49]. We believe these models can be generalised to the superspaces  $\mathbb{M}_T^{4|4}$ ,  $\mathbb{M}_S^{4|4}$  and  $\mathbb{M}_N^{4|4}$  to describe partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  supersymmetry breaking. It would also be interesting to investigate whether some of these models can be extended to describe partially broken  $\mathcal{N} = 2$  AdS supersymmetry.

Recently, there has been much interest in models for spontaneously broken local  $\mathcal{N} = 1$  supersymmetry [50, 51, 52, 53, 54, 55, 56, 57, 58], which are based on the use of the nilpotent chiral Goldstino superfield proposed in [59, 60]. Other nilpotent Goldstino superfields can be used to describe spontaneously broken  $\mathcal{N} = 1$  supergravity [61, 62, 63] (for an alternative approach to de Sitter supergravity, see [64]). At the moment it is not clear whether the nilpotent  $\mathcal{N} = 2$  chiral superfield advocated in the present paper is suitable for the description of partial supersymmetry breaking in  $\mathcal{N} = 2$  supergravity. It is certainly of interest to develop a superspace description for the models for spontaneous  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  local supersymmetry breaking pioneered in [65, 66] and further developed, e.g., in [67, 68].



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## A $\mathcal{N} = 1$ superspaces over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$ and $(AdS_3 \times S^1)/\mathbb{Z}_2$

In this appendix we give supermatrix realisations for two maximally supersymmetric backgrounds in 4D  $\mathcal{N} = 1$  supergravity.

### A.1 $\mathcal{N} = 1$ superspace over $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$

Here and in the next appendix, the supergroup  $SU(2|1)$  is defined to consist of complex  $(2|1) \times (2|1)$  supermatrices (with  $A, D$  bosonic blocks and  $B, C$  fermionic ones)

$$g = \left( \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \quad (A.1)$$

constrained by

$$g^\dagger \eta g = \eta, \quad \text{Ber } g = 1, \quad \eta = \left( \begin{array}{c|c} \mathbb{1}_2 & 0 \\ \hline 0 & -1 \end{array} \right). \quad (A.2)$$

We introduce a superspace  $\mathcal{M}^{4|4}$  consisting of complex  $(2|1) \times (2|0)$  supermatrices (with  $h$  bosonic and  $\Theta$  fermionic blocks)

$$\mathcal{P} = \left( \begin{array}{c} h \\ \hline \Theta \end{array} \right) \quad (A.3)$$

constrained by

$$\mathcal{P}^\dagger \eta \mathcal{P} = \mathbb{1}_2 \iff h^\dagger h = \mathbb{1}_2 + \Theta^\dagger \Theta. \quad (A.4)$$

The supermanifold defined by this equation coincides with the 4D  $\mathcal{N} = 1$  compactified Minkowski superspace (described in detail in section 3 of [69]) on which the superconformal group  $\text{SU}(2, 2|1)$  acts by well-defined transformations. The bosonic body of the superspace is  $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$ .

It is useful to switch from the variables  $h$  and  $\Theta$  to new ones,  $\varphi \in \mathbb{R}$ ,  $u$  and  $\theta$ , defined as follows:

$$\mathcal{P} = \begin{pmatrix} e^{i\varphi} u \\ e^{i\varphi} \theta \end{pmatrix}, \quad u^\dagger u = \mathbb{1}_2 + \theta^\dagger \theta, \quad \det u = \det u^\dagger = (1 + \theta \theta^\dagger)^{-\frac{1}{2}}. \quad (\text{A.5})$$

We can represent

$$u = \hat{u} \sqrt{\mathbb{1}_2 + \theta^\dagger \theta}, \quad \hat{u} \in \text{SU}(2). \quad (\text{A.6})$$

The supermatrix (A.5) is invariant under the  $\mathbb{Z}_2$  transformation  $\varphi \rightarrow \varphi + \pi$ ,  $\hat{u} \rightarrow -\hat{u}$  and  $\theta \rightarrow -\theta$ . This is the origin of  $\mathbb{Z}_2$  in  $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$ .

It turns out that the superspace  $\mathcal{M}^{4|4}$  introduced above can be identified with the group manifold  $\text{SU}(2|1)$ . Indeed, it may be checked that every element  $g \in \text{SU}(2|1)$  has the form (compare with a similar result in [70])

$$g = \left( \begin{array}{c|c} e^{i\varphi} u & e^{2i\varphi} (1 + \theta \theta^\dagger)^{-\frac{1}{2}} u \theta^\dagger \\ \hline e^{i\varphi} \theta & e^{2i\varphi} (1 + \theta \theta^\dagger)^{\frac{1}{2}} \end{array} \right), \quad (\text{A.7})$$

where  $u$  is constrained as in (A.5).

The isometry group of  $\mathcal{M}^{4|4}$  is  $\text{SU}(2|1) \times \text{U}(2)$ . It acts on  $\mathcal{M}^{4|4}$  as follows:

$$\mathcal{P} \rightarrow g_L \mathcal{P} g_R^{-1}, \quad g_L \in \text{SU}(2|1), \quad g_R \in \text{U}(2). \quad (\text{A.8})$$

These transformations are holomorphic in terms of the variables  $h$  and  $\Theta$  (hence the isometry transformations act on a chiral subspace of the full superspace). The isometry group has two  $\text{U}(1)$  subgroups that describe  $R$ -symmetry transformations and time translations. One subgroup corresponds to all diagonal supermatrices (A.7) with  $u = \mathbb{1}_2$  and  $\theta = 0$ . The other subgroup is spanned by all diagonal matrices  $e^{i\psi} \mathbb{1}_2$  in  $\text{U}(2)$ .

On the group manifold  $\text{SU}(2|1)$ , we can define an action of  $\text{SU}(2|1) \times \text{SU}(2|1)$  by the standard rule

$$g \rightarrow g_L g g_R^{-1}, \quad g_L, g_R \in \text{SU}(2|1). \quad (\text{A.9})$$

These transformations leave invariant the supermetric

$$ds^2 = -\frac{1}{2} \text{Str } \mathcal{E}^2, \quad \mathcal{E} = g^{-1} dg. \quad (\text{A.10})$$

However, such transformations map the chiral subspace (A.3) to itself only if  $g_R \in \text{U}(2)$ .

## A.2 $\mathcal{N} = 1$ superspace over $\mathbf{U}(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$

We define the supergroup  $\text{SU}(1, 1|1)$  to consist of complex  $(2|1) \times (2|1)$  supermatrices (with  $A, D$  bosonic blocks and  $B, C$  fermionic ones)

$$g = \left( \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \quad (\text{A.11})$$

constrained by

$$g^\dagger \eta g = \eta, \quad \text{Ber } g = 1, \quad \eta = \left( \begin{array}{c|c} \sigma_3 & 0 \\ \hline 0 & -1 \end{array} \right). \quad (\text{A.12})$$

Every element  $g \in \text{SU}(1, 1|1)$  can be written in the form

$$g = \left( \begin{array}{c|c} e^{i\varphi} u & e^{2i\varphi} (1 + \theta \sigma_3 \theta^\dagger)^{-\frac{1}{2}} u \theta^\dagger \\ \hline e^{i\varphi} \theta & e^{2i\varphi} (1 + \theta \sigma_3 \theta^\dagger)^{\frac{1}{2}} \end{array} \right), \quad (\text{A.13})$$

where  $u$  is constrained by

$$u^\dagger \sigma_3 u = \sigma_3 + \theta^\dagger \theta, \quad \det u = \det u^\dagger = (1 + \theta \sigma_3 \theta^\dagger)^{-\frac{1}{2}}. \quad (\text{A.14})$$

We can represent

$$u = \hat{u} \sqrt{\mathbb{1}_2 + \sigma_3 \theta^\dagger \theta}, \quad \hat{u} \in \text{SU}(1, 1). \quad (\text{A.15})$$

The supermatrix defined by eqs. (A.13) and (A.15) is invariant under the discrete transformation  $\varphi \rightarrow \varphi + \pi$ ,  $\hat{u} \rightarrow -\hat{u}$  and  $\theta \rightarrow -\theta$ .

We introduce a four-dimensional superspace  $\widetilde{\mathcal{M}}^{4|4}$  consisting of complex  $(2|1) \times (2|0)$  supermatrices (with  $h$  and  $\Theta$  being bosonic and fermionic blocks, respectively)

$$\mathcal{P} = \left( \begin{array}{c} h \\ \Theta \end{array} \right) \equiv \left( \begin{array}{c} e^{i\varphi} u \\ e^{i\varphi} \theta \end{array} \right), \quad (\text{A.16})$$

where  $\varphi$ ,  $u$  and  $\theta$  are defined as in (A.13). This superspace can be identified with the group manifold  $\text{SU}(1, 1|1)$ . Its bosonic body is  $\mathbf{U}(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$ .

The isometry group of  $\widetilde{\mathcal{M}}^{4|4}$  is  $\text{SU}(1, 1|1) \times \text{U}(2)$ . It acts on  $\widetilde{\mathcal{M}}^{4|4}$  as follows:

$$\mathcal{P} \rightarrow g_L \mathcal{P} g_R^{-1}, \quad g_L \in \text{SU}(1, 1|1), \quad g_R \in \text{U}(2). \quad (\text{A.17})$$

These transformations are holomorphic in terms of the variables  $h$  and  $\Theta$  (hence the isometry transformations acts on a chiral subspace of the full superspace), and leave invariant the supermetric

$$ds^2 = \frac{1}{2} \text{Str } \mathcal{E}^2, \quad \mathcal{E} = g^{-1} dg. \quad (\text{A.18})$$

Unlike the superspace considered in the previous subsection, the dimension parametrised by  $\varphi$  is now space-like.

Let us consider the coset space

$$\text{AdS}_{(3|2,0)} := \text{SU}(1, 1|1)/\text{U}(1), \quad (\text{A.19})$$

where the subgroup  $\text{U}(1)$  of  $\text{SU}(1, 1|1)$  consists of all diagonal supermatrices (A.13) with  $u = \mathbb{1}_2$  and  $\theta = 0$ . This coset space may be seen to coincide with the 3D (2,0) anti-de Sitter superspace [71]. We recall that in three dimensions,  $\mathcal{N}$ -extended anti-de Sitter (AdS) superspace exists in several incarnations known as  $(p, q)$  AdS superspaces, where the non-negative integers  $p \geq q$  are such that  $\mathcal{N} = p + q$ . The conformally flat  $(p, q)$  AdS superspace is

$$\text{AdS}_{(3|p,q)} = \frac{\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})}{\text{SL}(2, \mathbb{R}) \times \text{SO}(p) \times \text{SO}(q)}. \quad (\text{A.20})$$

In the case  $p = \mathcal{N} \geq 4$  and  $q = 0$ , non-conformally flat AdS superspaces also exist [72].

## B $\mathcal{N} = 2$ superspace over $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$

$\mathcal{N} = 2$  superspace  $\mathcal{M}^{4|8}$  over  $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$  can be realised as the quotient space

$$\mathcal{M}^{4|8} := \mathcal{M}_L^{4|4} \times \mathcal{M}_R^{4|4} / \sim, \quad (\text{B.1})$$

where  $\mathcal{M}_L^{4|4}$  and  $\mathcal{M}_R^{4|4}$  denote two copies of  $\mathcal{M}^{4|4}$ . The equivalence relation is defined by the rule: two pairs  $\mathcal{P} = (\mathcal{P}_L, \mathcal{P}_R)$  and  $\mathcal{P}' = (\mathcal{P}'_L, \mathcal{P}'_R)$  are equivalent,  $\mathcal{P} \sim \mathcal{P}'$ , if

$$\mathcal{P}'_L = \mathcal{P}_L h, \quad \mathcal{P}'_R = \mathcal{P}_R h, \quad (\text{B.2})$$

for some group element  $h \in \text{U}(2)$ .

The isometry group of  $\mathcal{M}^{4|8}$  is

$$\mathbf{G} := G_L \times G_R \times \text{U}(1) = \text{SU}(2|1) \times \text{SU}(2|1) \times \text{U}(1). \quad (\text{B.3})$$

Given a group element  $\mathbf{g} = g_L \times g_R \times e^{i\psi} \in \mathbf{G}$ , with  $\psi \in \mathbb{R}$ , it acts on the pair  $\mathcal{P} = (\mathcal{P}_L, \mathcal{P}_R)$  by the rule:

$$(\mathcal{P}_L, \mathcal{P}_R) \rightarrow (\mathbf{g}\mathcal{P}_L, \mathbf{g}\mathcal{P}_R), \quad \mathbf{g}\mathcal{P}_L = g_L \mathcal{P}_L e^{i\psi}, \quad \mathbf{g}\mathcal{P}_R = g_R \mathcal{P}_R e^{-i\psi}. \quad (\text{B.4})$$

The equivalence relation allows us to choose  $\mathcal{P}_R$  in the form:

$$\mathcal{P}_R = \begin{pmatrix} \sqrt{\mathbb{1}_2 + \psi^\dagger \psi} \\ \psi \end{pmatrix}. \quad (\text{B.5})$$

The above construction can readily be modified in order to describe the  $\mathcal{N} = 2$  super-space over  $U(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$ .

## C Example: The anti-de Sitter supersymmetry

In this appendix we show that the formalism of sections 2 and 3 can be used to define a Goldstone-Maxwell multiplet for partially broken 4D  $\mathcal{N} = 2$  anti-de Sitter (AdS) supersymmetry with the following properties: (i) it is the standard Maxwell multiplet with respect to the  $\mathcal{N} = 1$  AdS supersymmetry; (ii) it transforms nonlinearly under the second AdS supersymmetry. However, making use of this multiplet does not allow one to construct an invariant action describing the partial  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$  AdS supersymmetry breaking.

To start with, we recall a few definitions concerning the 4D  $\mathcal{N} = 2$  AdS superspace

$$\text{AdS}^{4|8} := \frac{\text{OSp}(2|4)}{\text{SO}(3, 1) \times \text{SO}(2)},$$

which is a maximally symmetric geometry that originates within the off-shell formulation for  $\mathcal{N} = 2$  conformal supergravity developed in [44]. For comprehensive studies of  $\mathcal{N} = 2$  supersymmetric field theories in  $\text{AdS}_4$ , the reader is referred to [73, 74].

We assume that  $\text{AdS}^{4|8}$  is parametrised by local bosonic  $(x)$  and fermionic  $(\theta, \bar{\theta})$  coordinates  $\mathbf{z}^{\mathcal{M}} = (x^m, \theta_\iota^\mu, \bar{\theta}_{\bar{\mu}}^{\bar{\iota}})$  (where  $m = 0, 1, 2, 3$ ,  $\mu = 1, 2$ ,  $\bar{\mu} = 1, 2$  and  $\iota = \underline{1}, \underline{2}$ ). The corresponding covariant derivatives

$$\mathcal{D}_{\mathcal{A}} = (\mathcal{D}_a, \mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\bar{\alpha}}^{\bar{i}}) = E_{\mathcal{A}}^{\mathcal{M}} \partial_{\mathcal{M}} + \frac{1}{2} \Omega_{\mathcal{A}}^{bc} M_{bc} + \Phi_{\mathcal{A}}^{ij} J_{ij}, \quad i, j = \underline{1}, \underline{2} \quad (\text{C.1})$$

obey the algebra [73]

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = 4S^{ij} M_{\alpha\beta} + 2\varepsilon_{\alpha\beta} \varepsilon^{ij} S^{kl} J_{kl}, \quad \{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_j^{\dot{\beta}}\} = -2i\delta_j^i (\sigma^c)_\alpha{}^{\dot{\beta}} \mathcal{D}_c, \quad (\text{C.2a})$$

$$[\mathcal{D}_a, \mathcal{D}_\beta^j] = \frac{i}{2}(\sigma_a)_{\beta\dot{\gamma}} S^{jk} \bar{\mathcal{D}}_k^{\dot{\gamma}} , \quad [\mathcal{D}_a, \bar{\mathcal{D}}_j^{\dot{\beta}}] = \frac{i}{2}(\tilde{\sigma}_a)^{\dot{\beta}\gamma} S_{jk} \mathcal{D}_\gamma^k , \quad (\text{C.2b})$$

$$[\mathcal{D}_a, \mathcal{D}_b] = -S^2 M_{ab} . \quad (\text{C.2c})$$

The  $\text{SU}(2)$  triplet  $S^{ij}$  is the only non-vanishing component of the superspace torsion in  $\text{AdS}^{4|8}$ ; it is *covariantly constant* and real

$$\mathcal{D}_A S^{ij} = 0 , \quad \bar{S}^{ij} = S^{ij} . \quad (\text{C.3})$$

The parameter  $S^2 := \frac{1}{2} S^{ij} S_{ij} = \text{const}$  is positive, and therefore (C.2c) gives the algebra of covariant derivatives in  $\text{AdS}_4$ .

The isometry transformations of  $\text{AdS}^{4|8}$  form the supergroup  $\text{OSp}(2|4)$ . In the infinitesimal case, an isometry transformation is described by a Killing supervector field  $\xi^A E_A$ , with  $E_A = E_A^{\mathcal{M}} \partial_{\mathcal{M}}$ , defined to obey the equation

$$\left[ \xi + \frac{1}{2} l^{bc} M_{bc} + \rho S^{jk} J_{jk}, \mathcal{D}_A \right] = 0 , \quad \xi := \xi^B \mathcal{D}_B = \xi^b \mathcal{D}_b + \xi_j^\beta \mathcal{D}_\beta^j + \bar{\xi}_{\dot{\beta}}^i \bar{\mathcal{D}}_{\dot{\beta}}^i , \quad (\text{C.4})$$

for some real antisymmetric tensor  $l^{bc}(z)$  and scalar  $\rho(z)$  parameters. It turns out that the Killing equation (C.4) uniquely determines the parameters  $\xi_i^\alpha$ ,  $l^{cd}$  and  $\rho$  in terms of  $\xi^a$ . A similar property exists for superspace isometry transformations in any number of dimensions [33]. The specific feature of the 4D  $\mathcal{N} = 2$  AdS superspace is that the parameters  $\xi^A$  and  $l^{ab}$  are uniquely expressed in terms of  $\rho$  [73].

Due to (C.2), the  $\text{SU}(2)$  gauge freedom can be used to choose the  $\text{SU}(2)$  connection  $\Phi_A^{ij}$  in (C.1) to look like  $\Phi_A^{ij} = \Phi_A S^{ij}$ , for some one-form  $\Phi_A$  describing the residual  $\text{U}(1)$  connection associated with the generator  $S^{ij} J_{ij}$ . Then  $S^{ij}$  becomes a constant iso-triplet,  $S^{ij} = \text{const}$ . The remaining global  $\text{SU}(2)$  rotations can take  $S^{ij}$  to any position on the two-sphere of radius  $S$ . We make the choice

$$S^{12} = 0 , \quad \mu := -S^{22} , \quad \bar{\mu} = -S^{11} , \quad (\text{C.5})$$

with  $|\mu| = S$ . This choice must be used in order to embed an  $\mathcal{N} = 1$  AdS superspace,  $\text{AdS}^{4|4}$ , into the full  $\mathcal{N} = 2$  AdS superspace [73].

As already mentioned, the choice  $S^{12} = 0$  is required for embedding  $\text{AdS}^{4|4}$  into  $\text{AdS}^{4|8}$ . By applying certain general coordinate and local  $\text{U}(1)$  transformations in  $\text{AdS}^{4|8}$ , it is possible to identify  $\text{AdS}^{4|4}$  with the surface  $\theta_2^\mu = 0$  and  $\bar{\theta}_\mu^2 = 0$ . The covariant derivatives for  $\text{AdS}^{4|4}$ ,

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}) = E_A^M \partial_M + \frac{1}{2} \Omega_A^{bc} M_{bc} , \quad (\text{C.6})$$

are related to (C.1) as follows

$$\mathcal{D}_\alpha := \mathcal{D}_\alpha^1|, \quad \bar{\mathcal{D}}^{\dot{\alpha}} := \bar{\mathcal{D}}_{\underline{1}}^{\dot{\alpha}}|, \quad (\text{C.7})$$

and similarly for the vector covariant derivative. Here the bar-projection is defined by

$$U| := U(x, \theta_i, \bar{\theta}^i)|_{\theta_2 = \bar{\theta}_2 = 0}, \quad (\text{C.8})$$

for any  $\mathcal{N} = 2$  tensor superfield  $U(x, \theta_i, \bar{\theta}^i)$ . It follows from (C.2) that the  $\mathcal{N} = 1$  covariant derivatives obey the algebra

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{\mu}M_{\alpha\beta}, \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 4\mu\bar{M}_{\dot{\alpha}\dot{\beta}}, \quad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i\mathcal{D}_{\alpha\dot{\beta}}, \quad (\text{C.9a})$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = -\frac{i}{2}\bar{\mu}(\sigma_a)_{\beta\dot{\gamma}}\bar{\mathcal{D}}^{\dot{\gamma}}, \quad [\mathcal{D}_a, \bar{\mathcal{D}}_{\dot{\beta}}] = \frac{i}{2}\mu(\sigma_a)_{\gamma\dot{\beta}}\mathcal{D}^\gamma, \quad (\text{C.9b})$$

$$[\mathcal{D}_a, \mathcal{D}_b] = -|\mu|^2 M_{ab}, \quad (\text{C.9c})$$

which indeed corresponds to the  $\mathcal{N} = 1$  AdS superspace (see [32] for more details). As a result, every  $\mathcal{N} = 2$  supersymmetric field theory in  $\text{AdS}^{4|8}$  can be reformulated as some theory in  $\text{AdS}^{4|4}$ .

Given an  $\mathcal{N} = 2$  tensor superfield  $U(x, \theta_i, \bar{\theta}^i)$ , its infinitesimal  $\text{OSp}(2|4)$  transformation law is

$$\delta_\xi U = -\left(\xi + \frac{1}{2}l^{bc}M_{bc} + \rho S^{jk}J_{jk}\right)U. \quad (\text{C.10})$$

Upon reduction to  $\text{AdS}^{4|4}$ , this transformation law turns into a superposition of several independent  $\mathcal{N} = 1$  transformations. Evaluating the bar-projection of  $\xi$  gives

$$\xi| = \lambda + \varepsilon^\alpha \mathcal{D}_\alpha^2| + \bar{\varepsilon}_{\dot{\alpha}} \bar{\mathcal{D}}_{\underline{2}}^{\dot{\alpha}}|, \quad \lambda = \lambda^A \mathcal{D}_A = \lambda^a \mathcal{D}_a + \lambda^\alpha \mathcal{D}_\alpha + \bar{\lambda}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}}, \quad (\text{C.11a})$$

where we have introduced

$$\lambda^a := \xi^a|, \quad \lambda^\alpha := \xi_{\underline{1}}^\alpha|, \quad \bar{\lambda}_{\dot{\alpha}} := \bar{\xi}_{\dot{\alpha}}^{\underline{1}}|, \quad \varepsilon^\alpha := \xi_{\underline{2}}^\alpha|, \quad \bar{\varepsilon}_{\dot{\alpha}} := \bar{\xi}_{\dot{\alpha}}^{\underline{2}}|. \quad (\text{C.11b})$$

We denote the bar-projection of the parameters  $l_{ab}$  and  $\rho$  as

$$\omega_{ab} := l_{ab}|, \quad \varepsilon := \rho|. \quad (\text{C.12})$$

It holds that

$$\omega_{\alpha\beta} = \mathcal{D}_\alpha \lambda_\beta = \mathcal{D}_\beta \lambda_\alpha. \quad (\text{C.13})$$

Now, the bar-projection of (C.10) takes the form

$$\delta_\xi U| = -\left(\lambda + \frac{1}{2}\omega^{ab}M_{ab}\right)U| - \left(\varepsilon^\alpha (\mathcal{D}_\alpha^2 U)| + \bar{\varepsilon}_{\dot{\alpha}} (\bar{\mathcal{D}}_{\underline{2}}^{\dot{\alpha}} U)|\right) + \varepsilon(\bar{\mu}J_{\underline{11}} + \mu J_{\underline{22}})U|. \quad (\text{C.14})$$

The first term on the right is an infinitesimal  $\text{OSp}(1|4)$  transformation generated by  $\lambda$ . The parameters  $\lambda$  and  $\omega^{bc}$  obey the equation

$$[\lambda + \frac{1}{2}\omega^{bc}M_{bc}, \mathcal{D}_A] = 0 , \quad (\text{C.15})$$

which defines the Killing supervector field of  $\text{AdS}^{4|4}$  [32]. The second and third terms on the right of (C.14) prove to describe the second supersymmetry and  $\text{U}(1)$  transformations. The corresponding parameters  $\varepsilon_\alpha$ ,  $\bar{\varepsilon}_{\dot{\alpha}}$  and  $\varepsilon$  have the properties

$$\varepsilon_\alpha = \frac{1}{2}\mathcal{D}_\alpha\varepsilon , \quad \mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot{\alpha}}\varepsilon = 0 , \quad (\mathcal{D}^2 - 4\bar{\mu})\varepsilon = 0 . \quad (\text{C.16})$$

The parameter  $\varepsilon$  was originally introduced in [75].

We are now prepared to analyse the nilpotent  $\mathcal{N} = 2$  chiral superfield  $\mathcal{Z}$  constrained by (2.1) in the case that the background superspace is  $\text{AdS}^{4|8}$ . We recall that a necessary ingredient of the construction described in section 3 is that  $G^{ij}$  is covariantly constant,  $\mathcal{D}_A G^{ij} = 0$ . We require this condition to hold in  $\text{AdS}^{4|8}$ , which implies that  $G^{ij}$  is proportional to  $S^{ij}$

$$G^{ij} = \kappa S^{ij} , \quad (\text{C.17})$$

where  $\kappa$  is a real constant. In accordance with (C.5), we have  $G^{12} = 0$ . The parameter  $\kappa$  can be chosen to have any given non-zero value by means of rescaling the chiral superfield  $\mathcal{Z}$ . We choose  $\kappa = |\mu|$ , and hence  $G^{11} = -|\mu|\bar{\mu}$  and  $G^{22} = -|\mu|\mu$ .

The degrees of freedom described by  $\mathcal{Z}$  are those of an Abelian  $\mathcal{N} = 1$  vector multiplet in  $\text{AdS}^{4|4}$ . Indeed, upon reduction to the  $\mathcal{N} = 1$  AdS superspace, the  $\mathcal{N} = 2$  chiral scalar  $\mathcal{Z}$  leads to two chiral superfields,  $X$  and  $W_\alpha$ , defined as

$$X := \mathcal{Z}| , \quad \bar{\mathcal{D}}_{\dot{\alpha}}X = 0 , \quad (\text{C.18a})$$

$$W_\alpha := -\frac{i}{2}\mathcal{D}_\alpha^2\mathcal{Z}| , \quad \bar{\mathcal{D}}_{\dot{\alpha}}W_\alpha = 0 . \quad (\text{C.18b})$$

One may check that the  $\mathcal{N} = 2$  constraints (2.1) imply the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} , \quad (\text{C.19a})$$

as well as the nonlinear constraint

$$-i\mu|\mu|X + \frac{1}{4}X(\bar{\mathcal{D}}^2 - 4\mu)\bar{X} = W^2 , \quad W^2 := W^\alpha W_\alpha . \quad (\text{C.19b})$$

Eq. (C.19a) tells us that  $W_\alpha$  is the chiral field strength of a Maxwell multiplet in  $\text{AdS}^{4|4}$ . Eq. (C.19b) is of the same type as the constraint (6.1), which generates the  $\mathcal{N} = 1$  locally



supersymmetric Born-Infeld action with  $U(1)$  duality invariance. The constraint (C.19b) is uniquely solved by expressing  $X$  in terms of  $W^2$  and  $\bar{W}^2$  and their covariant derivatives, in complete analogy with the general supergravity analysis of [38].

In accordance with (C.10), the infinitesimal  $O\text{Sp}(2|4)$  transformation of  $\mathcal{Z}$  is  $\delta\mathcal{Z} = -\xi\mathcal{Z}$ . Using this result, it is straightforward to derive the transformation laws of  $X$  and  $W_\alpha$  under the second supersymmetry and  $U(1)$  transformations described by the superfield parameter  $\varepsilon$ . Making use of the constraints obeyed by  $\mathcal{Z}$  and  $X$ , we obtain

$$\delta_\varepsilon X = -2i\varepsilon^\alpha W_\alpha , \quad (\text{C.20a})$$

$$\delta_\varepsilon W_\alpha = i\varepsilon_\alpha \left[ i\mu|\mu| - \frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar{X} - \mu X \right] - \bar{\varepsilon}^{\dot{\beta}} \mathcal{D}_{\alpha\dot{\beta}} X + \frac{i}{2}\mu\varepsilon \mathcal{D}_\alpha X . \quad (\text{C.20b})$$

One can check that  $\delta_\varepsilon X$  and  $\delta_\varepsilon W_\alpha$  preserve the constraints (C.19). Due to (C.16), the variation  $\delta_\varepsilon W_\alpha$  can be rewritten in the form

$$\delta_\varepsilon W_\alpha = -\frac{i}{8}(\bar{\mathcal{D}}^2 - 4\mu) \left[ 2(\bar{X} - X + i|\mu|) \varepsilon_\alpha - \varepsilon \mathcal{D}_\alpha X \right] , \quad (\text{C.20c})$$

which makes manifest the chirality of  $\delta_\varepsilon W_\alpha$ . It follows from (C.20) that the second supersymmetry and  $U(1)$  transformations are nonlinearly realised.

Let us consider the supersymmetric and  $U(1)$  duality invariant Born-Infeld action in the  $\mathcal{N} = 1$  AdS superspace<sup>10</sup>

$$S = -\frac{i}{4}|\mu|\mu \int d^4x d^2\theta \mathcal{E} X + \text{c.c.} , \quad (\text{C.21})$$

with  $X$  constrained by (C.19b). The action is manifestly invariant under the isometry transformations of  $\text{AdS}^{4|4}$ , with the infinitesimal transformation law of  $W_\alpha$  being

$$\delta W_\alpha = -\lambda W_\alpha - \omega_\alpha{}^\beta W_\beta . \quad (\text{C.22})$$

However, the action is not invariant under the transformation (C.20),

$$\delta_\varepsilon S = 2|\mu|^3 \int d^4x d^2\theta d^2\bar{\theta} E \varepsilon V , \quad (\text{C.23})$$

where the real scalar  $V$  denotes the unconstrained prepotential of the vector multiplet,

$$W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}_\alpha V . \quad (\text{C.24})$$

Eq. (C.23) is a unique feature that distinguishes  $\text{AdS}_4$  from the other maximally supersymmetric backgrounds we have studied in this paper.

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<sup>10</sup>In accordance with (C.19b), the overall coefficient in (C.21) is chosen such that the kinetic term for the vector multiplet is canonically normalised,  $S = \frac{1}{4} \int d^4x d^2\theta \mathcal{E} W^2 + \text{c.c.} + \text{interaction terms}$ . It should be remarked that the functional  $\text{Re}(\mu \int d^4x d^2\theta \mathcal{E} X)$  is a total derivative, in accordance with (C.19b).

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